Delft University of Technology

Project: Parallel Multiplicative One-Level Schwarz Preconditioners With FROSch and Trilinos

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May 11, 2021

Project Description

Schwarz methods are an algorithmic framework for a large class of domain decomposition methods. The **software FROSch (Fast and Robust Overlapping Schwarz) [1], which is part of the Trilinos [2] package ShyLU**, provides a **highly scalable MPI-parallel implementation of the Schwarz framework**, and the resulting solvers are based on the construction and combination of the relevant Schwarz operators. FROSch currently focusses on Schwarz operators that are algebraic in the sense that they can be constructed from a fully assembled, parallel distributed matrix.

Consider the linear equation system

$$Ku = f$$

arising from the discretization of a boundary value problem. With respect to the first level of parallel Schwarz preconditioners, FROSch currently only uses additive variants of the form

$$P = M^{-1}K = \sum_{i=1}^{N} \tilde{R}_i^T K_i^{-1} R_i K. \tag{1}$$

Here, R_i, $\tilde{R}_i^T$, and K_i correspond to restriction and prolongation operators and the local stiffness matrix corresponding to the ith overlapping subdomain, respectively. This is advantageous because each term $\tilde{R}_i^T K_i^{-1} R_i$ can be computed independently in parallel.

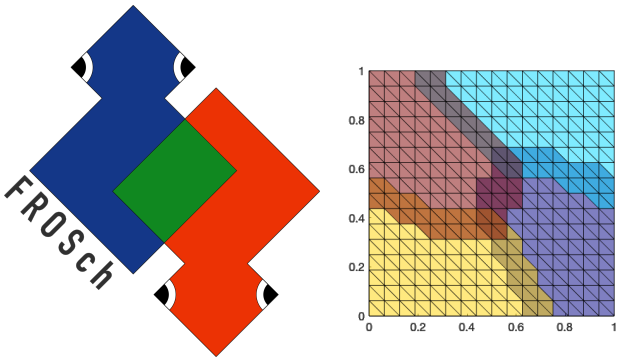

Figure 1: FROSch logo (left) and unstructured overlapping domain decomposition into four overlapping subdomains with one layer of overlap.

The goal of this project is to instead **implement parallel multiplicative one-level Schwarz operators** of the form

$$P = I - \prod_{i=1}^{N} (I - P_i) \tag{2}$$

using the building blocks of the additive one-level Schwarz implementation; see also [3, Sect. 2.2].

This project will be part of a **collaboration with the Scalable Algorithms group at the Sandia National Laboratories**[1] (SNL), United States, on the development of the FROSch solver framework. The Sandia National Laboratories are the main developers of Trilinos.

Tasks

- Read [3, Sect. 1.4 and Chapt. 2].
- Install and familiarize with the Trilinos[2] software library and, in particular, with the[3] code; see the Trilinos GitHub repository[4] for the code.
- Implement fixed point iteration with a multiplicative Schwarz operator and sequential subdomain solves.
- Employ coloring techniques (e.g., using the Trilinos package Zoltan2) to parallelize the multiplicative Schwarz operator.
- Comparison of the parallel scalability of the new implementation against the state of the art one-level Schwarz preconditioners in FROSch.

Contact

If you are interested in this project and/or have further questions, please contact Alexander Heinlein, a.heinlein@tudelft.nl.

[1] https://www.sandia.gov
[2] https://trilinos.github.io
[3] https://shylu-frosch.github.io
[4] https://github.com/trilinos/Trilinos

References

[1] A. Heinlein, A. Klawonn, S. Rajamanickam, and O. Rheinbach. FROSch: A fast and robust overlapping Schwarz domain decomposition preconditioner based on Xpetra in Trilinos. In *Domain Decomposition Methods in Science and Engineering XXV*, pages 176–184. Springer International Publishing, 2020. doi: 10.1007/978-3-030-56750-7_19. URL https://link.springer.com/chapter/10.1007/978-3-030-56750-7_19.

[2] M. A. Heroux, R. A. Bartlett, V. E. Howle, R. J. Hoekstra, J. J. Hu, T. G. Kolda, R. B. Lehoucq, K. R. Long, R. P. Pawlowski, E. T. Phipps, A. G. Salinger, H. K. Thornquist, R. S. Tuminaro, J. M. Willenbring, A. Williams, and K. S. Stanley. An overview of the Trilinos project. *ACM Trans. Math. Softw.*, 31(3): 397–423, 2005. ISSN 0098-3500. doi: http://doi.acm.org/10.1145/1089014.1089021.

[3] A. Toselli and O. Widlund. *Domain decomposition methods—algorithms and theory*, volume 34 of *Springer Series in Computational Mathematics*. Springer-Verlag, Berlin, 2005. ISBN 3-540-20696-5.