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Project: Overlapping Schwarz Domain Decomposition Methods for Implicit Ocean Models

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Project Description

Earth system models (ESMs) are one of the key players in making projections for the **future development of the Earth's climate**. An important component of these models are **ocean models that simulate the flow of sea water based on physical conservation laws** (conservation of mass, momentum and energy). These laws are formulated as partial differential equations known as the 'primitive equations': they describe fluid flow in a rotating, spherical domain under certain assumptions (e.g. an incompressible fluid).

Practically all ocean models in use today rely on explicit time integration schemes. For increasing mesh resolutions, time step constraints due to the Courant–Friedrich–Levy (CFL) condition become increasingly strict. Therefore, for simulating long time spans or to compute a (quasi) stationary state, it may become desirable to use implicit time integration schemes, such that efficient numerical solvers for large linear equations systems are needed. Possible applications are scientific discovery (e.g. when simulating the climate of past times in paleo-climate research) or computing a starting configuration for virtual experiments ('spin-up problem').

In the past, we have shown that **efficient simulations** with implicit time stepping without restrictions on the step size can be performed using **tailored preconditioning techniques and high performance computing (HPC)** [6]. Here, nested iterative methods were employed to split the problem into simpler linear systems. This leads to a complex convergence behavior and large overall iteration counts.

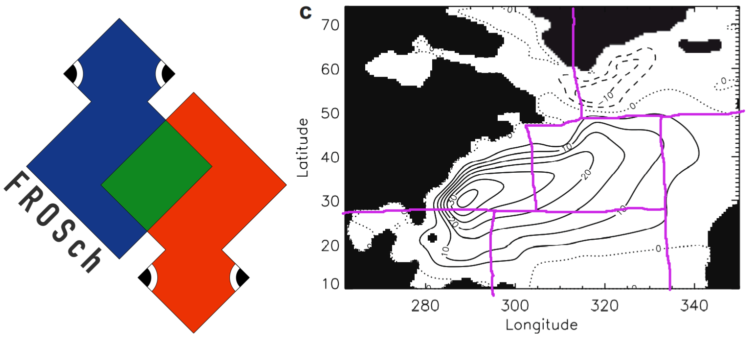

Figure 1: FROSch logo (left) and stationary solution of the ocean equations in the northern Atlantic ocean computed by the fully implicit ocean model THCM (right); cf. [6, Fig 5c]. The lines indicate a fictive domain decomposition for parallel processing.

The goal of this project is to take a new route and employ a **fully coupled preconditioning approach based on monolithic overlapping Schwarz domain decomposition methods** [2] using the **FROSch (Fast and Robust Overlapping Schwarz) [3] solver framework**, which is part of Trilinos [4]. As the basis for the simulations, the **Trilinos-based implementation of the implicit Earth system model of intermediate complexity (I-EMIC)**[1] is used; see [1, 5] for the model equations.

The project builds on the current **state of the art in implicit Earth system modeling and solver technology**. It deals with an open research question with potentially significant impact. Depending on the findings and the candidate's enthusiasm, a subsequent publication of the results in a peer-reviewed journal is an option. During the project, a direct **collaboration with climate scientists who developed the I-EMIC model at the Institute for Marine and Atmospheric Modeling at Utrecht University** is possible and is also aspired.

Tasks

- Install and familiarize with:
  - the Trilinos[2] software library and, in particular, with the FROSch[3] code; see the Trilinos GitHub repository[4].
  - the I-EMIC parallel Earth system model software; see the GitHub repository[5] for the code.
  - the respective literature: the physical equations and their discretization as well as with domain decomposition methods, in particular, monolithic Schwarz methods.
- Experiment with the I-EMIC model and the FROSch preconditioner and, if necessary, implement modifications in the software interface and the preconditioner.
- Perform parallel simulations to asses the numerical behavior and the parallel performance. Then, the observations should be matched with the theory.

[1] https://github.com/nlesc-smcm/i-emic
[2] https://trilinos.github.io
[3] https://shylu-frosch.github.io
[4] https://github.com/trilinos/Trilinos
[5] https://github.com/nlesc-smcm/i-emic

Contact

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References

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