Delft University of Technology

# Project: Error Estimates for Finite Element Simulations Using Neural Networks

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# Project Description

**Adaptive finite element methods** are based on the observation that **the error** between the discrete finite element solution and the exact solution **is not uniformly distributed** over the computational domain. For instance, if, due to a loss of regularity, the solution has a singularity, the **error is much higher in the vicinity of the singularity**; see fig. 1.

It is well known that the **accuracy of the discrete solution can be improved by refining the mesh**. However, refining the mesh uniformly will reduce the error in all parts of the domain, even in those areas where the error is already sufficiently small. In order to **improve the efficiency**, the mesh should therefore only be **refined in areas with a high error**. Since the exact solution and the exact error are typically unknown, a local error estimator is necessary to identify the regions where the mesh has to be refined.

Based on the this, an adaptive finite element strategy comprises three components:

- A **finite element implementation** for a given mesh
- An **a posteriori error estimator**, which estimates the error of a discrete solution; see, e.g., [2]
- A **mesh refinement strategy**

The adaptive finite element approach is then essentially a recursive loop in which the **mesh is refined locally where the error estimator indicates a high error of the discrete solution**. This procedure is **repeated until the error estimate is below a certain tolerance**. In fig. 1, we can observe that the error can already be reduced significantly by performing two steps of local mesh refinement.

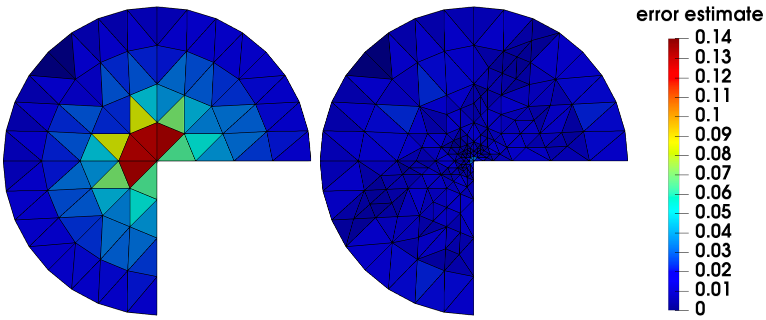

Figure 1: Adaptive mesh refinement on a *PAC-MAN* geometry: residual based a posteriori error estimate on a uniform mesh and after two steps of adaptive mesh refinement.

Obviously, the efficiency of the adaptive finite element strategy is particularly determined by the accuracy of the a posteriori error estimator and the mesh refinement strategy. Therefore, the goal of this master project is to **investigate if**, based on a discrete finite element solution, **machine learning techniques, such as neural networks, can be used to indicate which elements should be refined**; see [1] for more details on neural networks.

# Tasks

- Install and familiarize with the software packages:
  - The finite element software FEniCS[1].
  - The Python machine learning libraries TensorFlow 2.0[2] and Keras[3].
- Familiarize with adaptive mesh refinement for finite element methods, and implement adaptive finite elements for simple model problems using FEniCS.
- Familiarize with the basic concepts of neural networks.
- Train a neural network in order to predict which elements should be refined in the adaptive finite element implementation.
- Combine the adaptive finite element implementation and the neural network and perform numerical studies to investigate the efficiency of the approach, compared to the initial adaptive finite element implementation.

[1] https://fenicsproject.org
[2] https://www.tensorflow.org
[3] https://keras.io

## Contact

If you are interested in this project and/or have further questions please contact Alexander Heinlein, a.heinlein@tudelft.nl, and Deepesh Toshniwal, d.toshniwal@tudelft.nl.

## References

[1] I. Goodfellow, Y. Bengio, and A. Courville. *Deep learning*. MIT press, 2016.

[2] R. Verfürth. *A Posteriori Error Estimation Techniques for Finite Element Methods*. Numerical Mathematics and Scientific Computation. OUP Oxford, 2013.