

The Application of Neural Networks to Predict Skin Evolution After Burn Trauma

Literature Presentation



Selma Husanovic

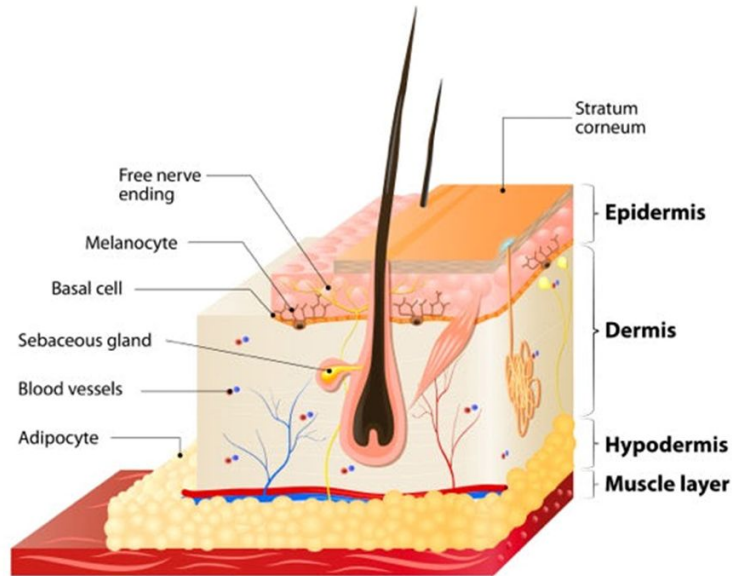
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Biological Background

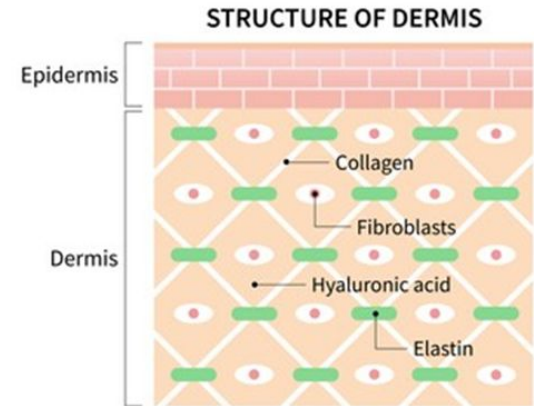
Skin Consists of Three Layers



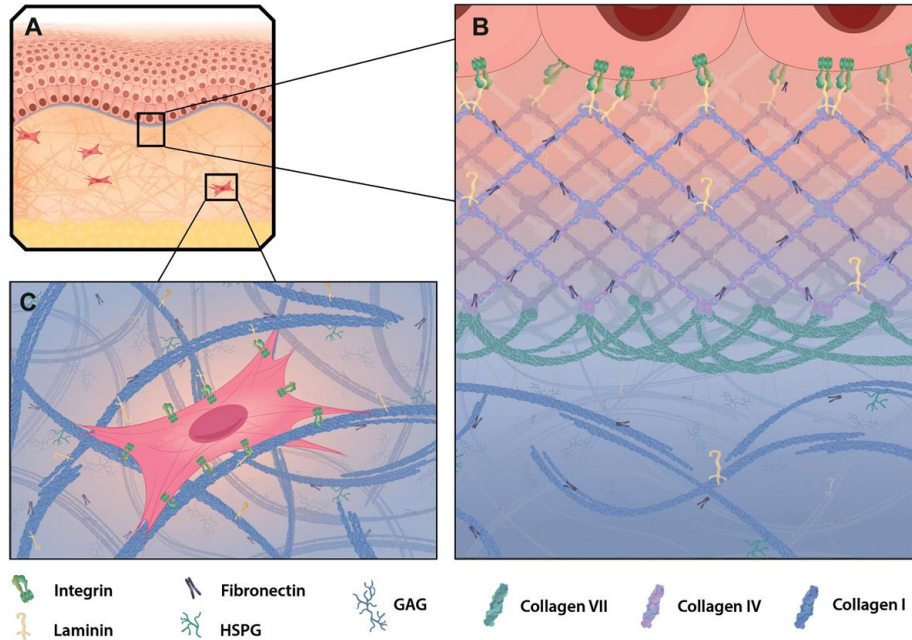
- Wounds to the epidermis are superficial and heal quickly, without scarring
- Wounds that affect the dermis are more complex, as the damaged dermis needs to be regenerated
- The focus is on the latter

The Dermis Consists of

- Water
- Cells: **fibroblasts** being the most common ones
- ECM: 3D network of molecules that surrounds and supports the cells. Consist mostly out of **collagen** proteins

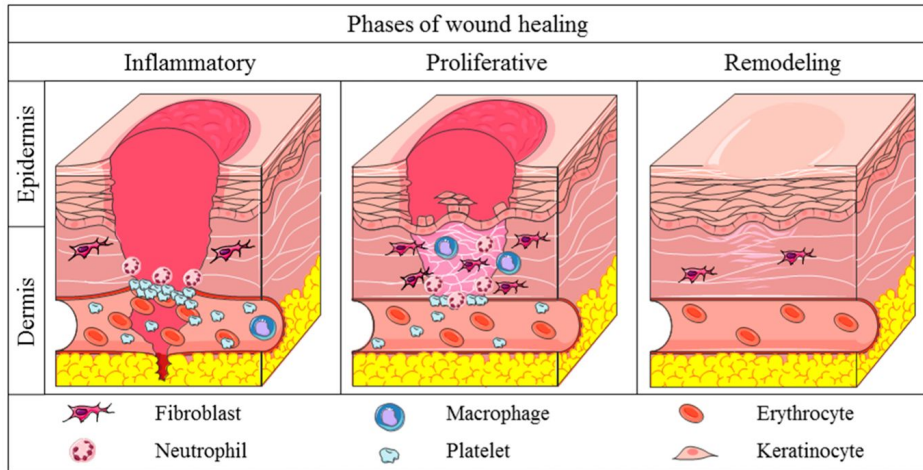


Extracellular Matrix (ECM)



- Acts as a scaffold for cells, such that they can migrate and perform their functioning
- Contains **signalling molecules**

Wound Healing (ECM is damaged)



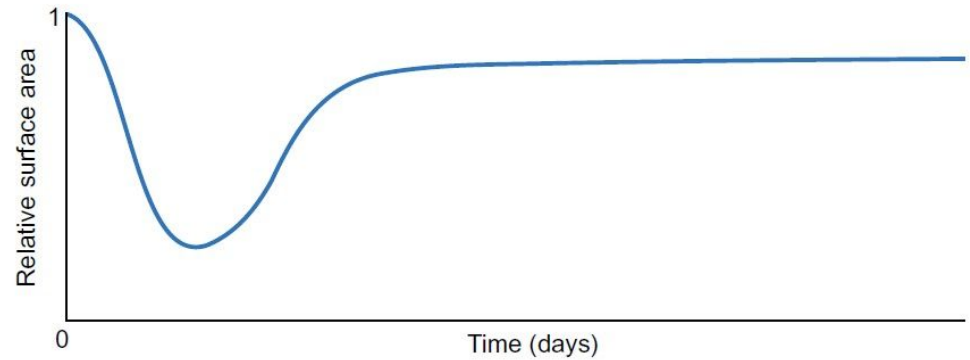
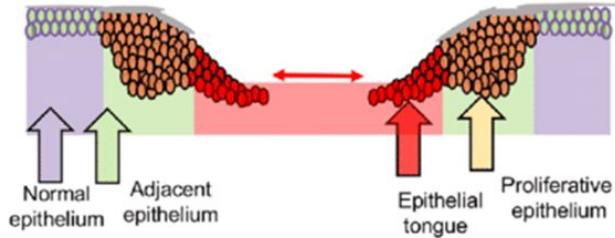
- Fibrin cloth is made, acting as temporary ECM
- Pathogens are removed
- Fibroblasts come in and deposit new ECM
- Signalling molecules:
fibroblasts -> myofibroblasts
- Myofibroblasts pull on the ECM
- New ECM undergoes reorganisation

Contraction

b Day 1



c Day 4





Mechanical Background

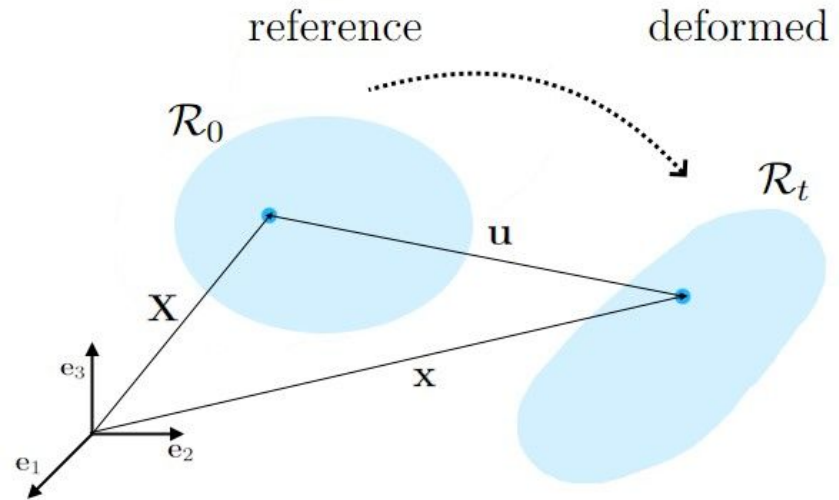
Description of Motion

Lagrangian / Material Description:

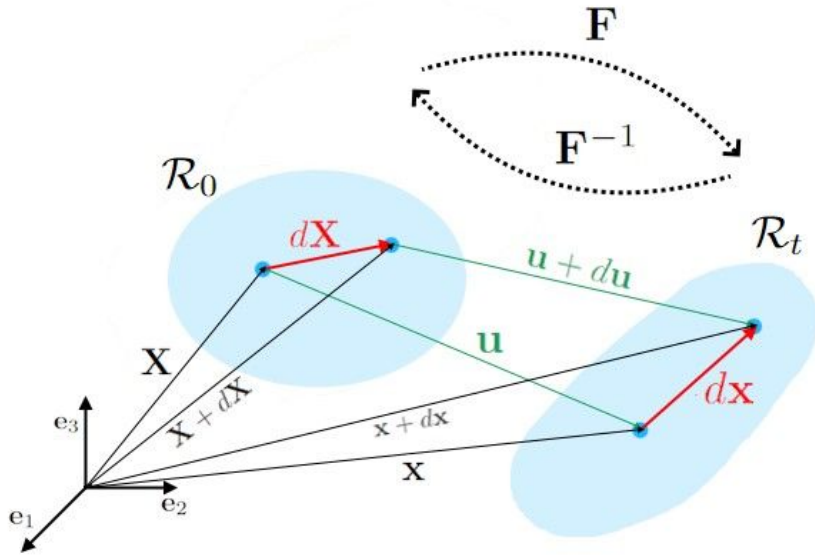
$$\mathbf{x} = \mathbf{f}(\mathbf{X}, t)$$

Eulerian / Spatial Description:

$$\mathbf{X} = \mathbf{f}^{-1}(\mathbf{x}, t)$$



Deformation Gradient Tensor



- Fundamental measure of deformation
- Maps line elements in reference configuration to line elements in current configuration

$$d\mathbf{x} = \mathbf{F} d\mathbf{X}$$

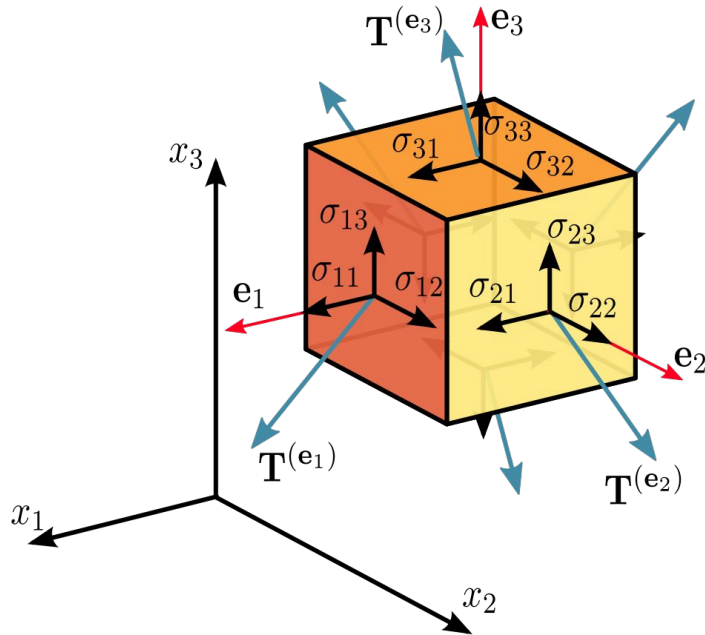
Strain tensor

- Strain is a measure of deformation of a body with respect to a reference configuration
- Presence of a stress will generally lead to strain

$$\boldsymbol{\varepsilon} = \frac{1}{2} \left[\nabla \mathbf{u} + (\nabla \mathbf{u})^T \right]$$

$$\boldsymbol{\varepsilon} = \begin{pmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{pmatrix}$$

Stress tensor



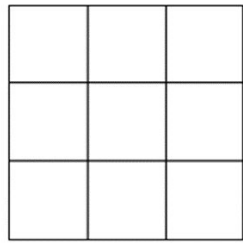
- Stress expresses the internal forces that neighbouring material particles exert on each other
- This can be as a reaction to external forces on the surface

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix}$$



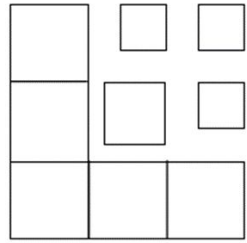
Morphoelastic Model for Burn Injuries

Morphoelasticity

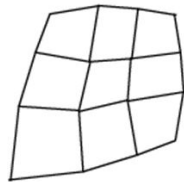


Initial Configuration

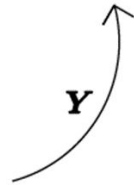
$$\mathbf{Z} \equiv \mathbf{F}_g$$



Zero Stress State



Current Configuration



$$\mathbf{Y} \equiv \mathbf{F}_e^{-1}$$

Decomposition:

$$\begin{aligned} \mathbf{F} &= \frac{\partial \mathbf{x}}{\partial \mathbf{X}} \\ &= \frac{\partial \mathbf{x}}{\partial \boldsymbol{\chi}} \frac{\partial \boldsymbol{\chi}}{\partial \mathbf{X}} = \mathbf{F}_e \mathbf{F}_g \end{aligned}$$

Mathematical Model

$$\frac{Dz_i}{Dt} + z_i(\nabla \cdot \mathbf{v}) = -\nabla \cdot \mathbf{J}_i + R_i,$$

conservation of cell density / concentration

$$\rho_t \left(\frac{D\mathbf{v}}{Dt} + \mathbf{v}(\nabla \cdot \mathbf{v}) \right) = \nabla \cdot \boldsymbol{\sigma} + \mathbf{f},$$

conservation of linear momentum

$$\frac{D\boldsymbol{\varepsilon}}{Dt} + \boldsymbol{\varepsilon} \text{skw}(\nabla \mathbf{v}) - \text{skw}(\nabla \mathbf{v}) \boldsymbol{\varepsilon} + (\text{tr}(\boldsymbol{\varepsilon}) - 1) \text{sym}(\nabla \mathbf{v}) = -\mathbf{G}.$$

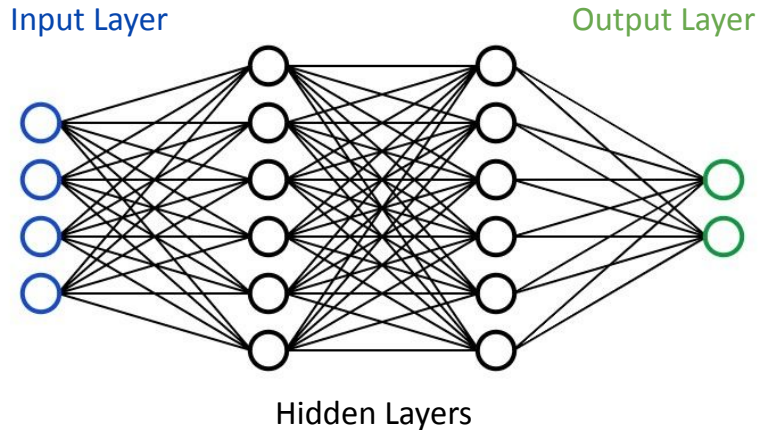
evolution equation of strain, morphoelasticity of dermal layer

Biological Constituents		Mechanical Components	
N	fibroblasts	\mathbf{C}	signalling molecule
M	myo-fibroblasts	ρ	collagen
		\mathbf{u}	displacement
		$\boldsymbol{\varepsilon}$	strain
		\mathbf{v}	velocity



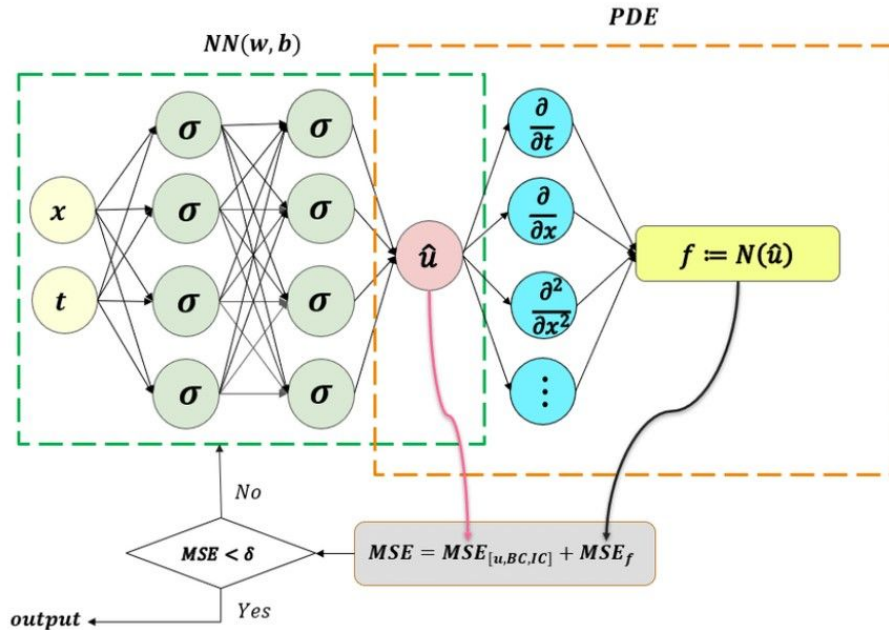
Machine Learning Background

Neural Networks (NNs)



- Layers of interconnected nodes (neurons)
- Each neuron receives input. Inputs are weighted, summed and passed through an activation function to produce the neuron's output
- NN minimises a loss function by adjusting weights and biases during training

Physics Informed Neural Networks (PINNs)



Use the physical prior knowledge (PDEs) to constrain the NN's space of admissible solutions

$$u_t + \mathcal{N}[u; \lambda] = 0, \quad x \in \Omega, t \in [0, T]$$

$$f := u_t + \mathcal{N}[u; \lambda]$$

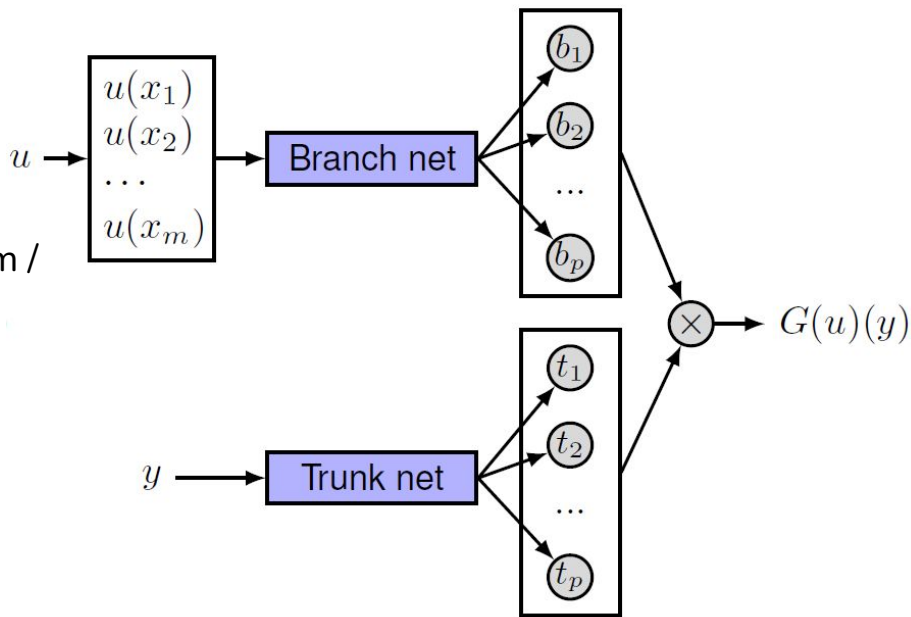
$$MSE = MSE_u + MSE_f$$

$$MSE_u = \frac{1}{N_u} \sum_{i=1}^{N_u} |u(t_u^i, x_u^i) - u^i|^2,$$

$$MSE_f = \frac{1}{N_f} \sum_{i=1}^{N_f} |f(t_f^i, x_f^i)|^2.$$

Deep Operator Networks (DeepONets)

- Learns maps between function spaces:
solution operator of PDE
- There is some input parameter \mathbf{u} the solution depends on. Could be forcing term / source term / IC / coefficient / domain geometry
- Learn the map between \mathbf{u} and the whole spatio-temporal solution





What Has Been Done?

ML for Wound Healing

- 1D model: simple feedforward NN with 25 parameters as inputs and RSAW, displacement vector and strain energy as output
- Same for 2D model
- Fixed wound shape (rotated square in 2D) and data generated using the FEM model
- Hybrid model: NN as surrogate for computationally expensive step in FEM for solving 1D problem



Direction of Research

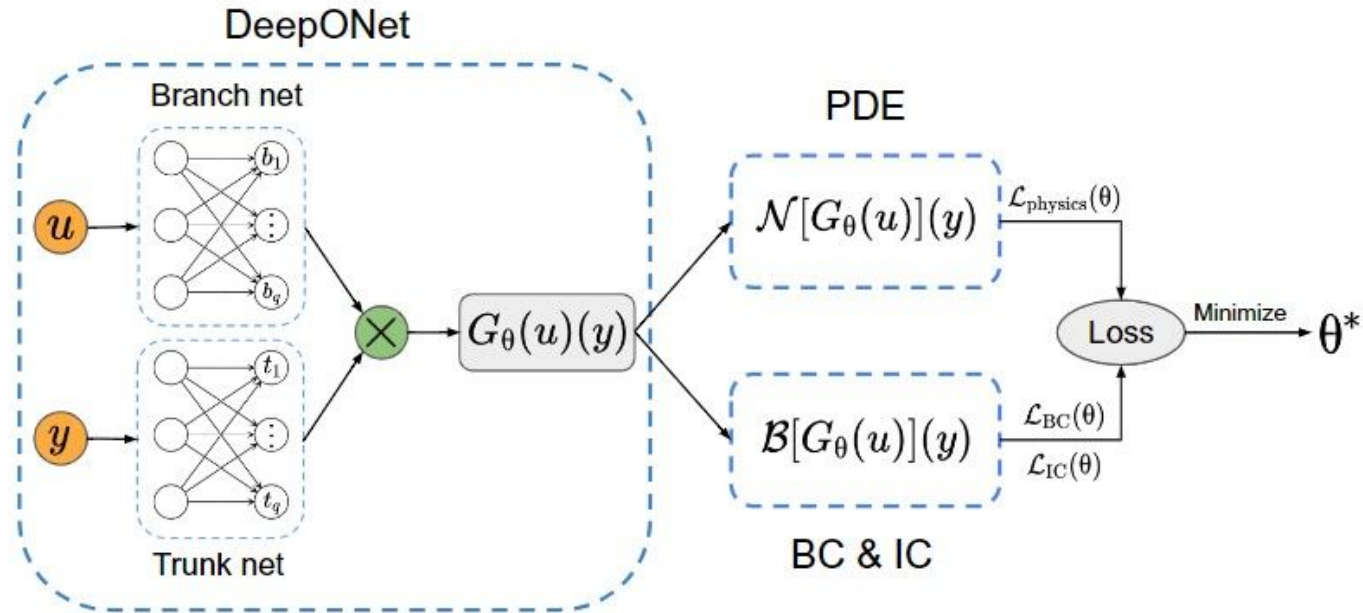
Research Question

Can a neural network be trained to predict the entire time evolution of skin contraction, given only the initial geometry of the wound as input?

For this we would incorporate:

- DeepONet
- PINNs setup in the loss function
- Data using the finite element simulation

Physics Informed DeepONets



Approach

- Start simple
- Given one material parameter, we want to predict the displacement of the wound at fixed time T
- If this is feasible using the proposed set-up, use two material parameters
- Consider the input to the branch net to be a continuous function
- Consider the input to the branch net to be discrete (image of the wound)



Thank you for your
attention!



Complete Model

$$\begin{aligned} \frac{DN}{Dt} + N(\nabla \cdot \mathbf{v}) &= -\nabla \cdot (-D_F F \nabla N + \chi_F N \nabla c) + r_F \left(1 + \frac{r_F^{\max} c}{a_c^I + c} \right) (1 - \kappa_F F) N^{1+q} - k_F c N - \delta_N N, \\ \frac{DM}{Dt} + M(\nabla \cdot \mathbf{v}) &= -\nabla \cdot (-D_F F \nabla M + \chi_F M \nabla c) + r_F \left(\frac{(1 + r_F^{\max}) c}{a_c^I + c} \right) (1 - \kappa_F F) M^{1+q} - k_F c M - \delta_M M, \\ \frac{Dc}{Dt} + c(\nabla \cdot \mathbf{v}) &= -\nabla \cdot (-D_c \nabla c) + \frac{k_c (N + \eta^I M) c}{a_c^{II} + c} - \delta_c \frac{(N + \eta^{II} M) \rho}{1 + a_c^{III} c} c, \\ \frac{D\rho}{Dt} + \rho(\nabla \cdot \mathbf{v}) &= k_\rho \left(1 + \frac{k_\rho^{\max} c}{a_c^{IV} + c} \right) (N + \eta M) - \delta_\rho \frac{(N + \eta^{II} M) \rho}{1 + a_c^{III} c} \rho, \\ \rho_t \left(\frac{D\mathbf{v}}{Dt} + \mathbf{v}(\nabla \cdot \mathbf{v}) \right) &= \nabla \cdot \left(\mu_1 \text{sym}(\nabla \mathbf{v}) + \mu_2 [\text{tr}(\text{sym}(\nabla \mathbf{v})) \mathbf{I}] \right) + \frac{E \sqrt{\rho}}{1 + \nu} \left(\boldsymbol{\varepsilon} + \text{tr}(\boldsymbol{\varepsilon}) \frac{\nu}{1 - 2\nu} \mathbf{I} \right) + \nabla \cdot \left(\xi M \left(\frac{\rho}{R^2 + \rho^2} \right) \mathbf{I} \right), \\ \frac{D\boldsymbol{\varepsilon}}{Dt} + \boldsymbol{\varepsilon} \text{skw}(\nabla \mathbf{v}) - \text{skw}(\nabla \mathbf{v}) \boldsymbol{\varepsilon} + (\text{tr}(\boldsymbol{\varepsilon}) - 1) \text{sym}(\nabla \mathbf{v}) &= -\xi \left(\frac{(N + \eta^{II} M) c}{1 + a_c^{III} c} \right) \boldsymbol{\varepsilon}. \end{aligned}$$