Project: Domain Decomposition Techniques for the Helmholtz Equation – Theoretical Investigation

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Our group is world leading in the development of fast and robust solvers for discretized Helmholtz problems. The Helmholtz equation is the time-harmonic equivalent of the wave equation and is used in a wide range of engineering practices. It mainly models electromagnetic waves and is used from scattering studies in medical imaging to seismology.

Several issues arise when trying to solve the Helmholtz equation numerically. One of them is related to the efficiency of the numerical solvers. Due to the size and indefiniteness of the linear system, numerical solutions are obtained by using iterative solvers, in particular GMRES. Moreover, depending on the boundary conditions, the matrix becomes complex non-Hermitian. The challenge in designing efficient solvers is that the number of iterations to reach convergence grows with the wavenumber.

In our group, recent research has focused on developing a domain decomposition preconditioner for the Helmholtz equation, in particular a two-level Schwarz preconditioner that has wavenumber independent convergence and has reduced computational costs. One important feature here was the introduction of first-order or a higher-order Bézier interpolation to construct the coarse-spaces [1, 2]. For a simple model problem, the convergence of the iterative method can be deduced by analyzing the spectrum and eigenvalues. However, it seemed that even for the simple model, the convergence behavior could not be fully explained by the spectral analysis [3]. This MSc. project is designed to figure out why this is the case by comparing the convergence behavior and spectrum of the first-order and second-order interpolation schemes for the coarse spaces.

At the end of the project, the aim is to have developed a novel theoretical framework to understand and explain the convergence behavior of the numerical solver.The results will be crucial to the further development of scalable and accurate solvers for indefinite Helmholtz problems.

1 Research plan and schedule

In this section we outline the research plan and schedule.

- Prerequisites: real and complex analysis, linear algebra, scientific computing, familiarity with programming in Matlab or Python.
- Literature study (approx. 1 month) with topics:
  - Helmholtz equation
  - GMRES iterative method
  - Interpolation method
  - Domain decomposition method (in particular two-level Schwarz methods)
- Research phase I: Preliminary implementation for 1D/2D simple model problem using two-level Schwarz and first- and second-order interpolation schemes (approx. 2-3 months).
- Research phase II: Spectral analysis, detailed comparison of both methods including location of smallest eigenvalues and shifts (approx. 2-3 months).
- Research phase III: Finalizing thesis (approx. 1 month).

Contact

If you are interested in this project and/or have further questions please contact Vandana Dwarka, v.n.s.r.dwarka@tudelft.nl, and Alexander Heinlein, a.heinlein@tudelft.nl.

References

[1] V. Dwarka and C. Vuik. Scalable convergence using two-level deflation preconditioning for the helmholtz equation. *SIAM Journal on Scientific Computing*, 42(2):A901–A928, 2020.

[2] A. Heinlein, A. Klawonn, S. Rajamanickam, and O. Rheinbach. *FROSch: A Fast And Robust Overlapping Schwarz Domain Decomposition Preconditioner Based on Xpetra in Trilinos*, pages 176–184. Springer, 2020.

[3] Erik Sieburgh. Domain decomposition helmholtz solvers: Obtaining wave number independence. 2022.