MSc project: Numerical techniques for efficiently solving a nonlinear model for salt intrusion in rivers

Where rivers meet the sea, salty sea water can intrude upriver for anything between 1 and 100 km. Due to sea level rise and extreme droughts, this upriver intrusion of salt water is increasing in rivers Worldwide. In many rivers, this is increasingly threatening for drinking water supply and agriculture, which rely on fresh water. Mathematical modelling of salt intrusion can help to understand and predict such salt intrusion problems.

Mathematically, salt intrusion is described by a set of coupled nonlinear partial differential equations describing the flow of water and transport of salt. As no analytical solutions are possible, numerical methods are used. However, due to the strong non-linearity, accurate numerical solutions are not so easily found. If one naively applies basic numerical methods, one may obtain systems which are badly conditioned, non-converging, oscillating and not efficiently solvable. To solve this, it is time to use and expand your skills in numerical mathematics!

In this project you will start with a simplified model for salt intrusion. Combining knowledge of the physics and numerical methods, you will develop a robust and accurate numerical implementation for this model. You will work with supervisors with expertise on both the modelling/physics as well as numerical techniques. We will explore various cutting-edge numerical methods for solving the linear and nonlinear equation systems. This includes nonlinear preconditioning techniques based on nonlinear elimination and linear/nonlinear domain decomposition methods.

Depending on your interests, you may continue the project by adding more physical processes to the model, focusing more deeply on the various numerical techniques or focusing on high-performance computing.

Requirements
This projects is suitable for students from various tracks in the Applied Mathematics program interested in applied numerical mathematics. Useful (but not mandatory) courses from the MSc program include Scientific Computing and Computational Fluid Dynamics.

Supervision
This work is a joint project between the groups of Numerical Analysis (NA) and Mathematical Physics (MP) and you will be supervised by Alexander Heinlein (NA) and Yoeri Dijkstra (MP).

Interested: send an email to Alexander Heinlein (a.heinlein@tudelft.nl) or Yoeri Dijkstra (y.m.dijkstra@tudelft.nl).

Dijkstra, Y. M., Schuttelaars, H. M., Kranenburg, W. M. (2022). Salt Transport Regimes Caused by Tidal and Subtidal Processes in Narrow Estuaries. Journal of Geophysical Research: Oceans, 127:e2021JC018391.

Cresta, P., Allix, O., Rey, C., and Guinard, S. (2007). Nonlinear localization strategies for domain decomposition methods: Application to post-buckling analyses. Computer Methods in Applied Mechanics and Engineering, 196(8), 1436-1446.

Heinlein, A. and Lanser, M. (2020). Additive and hybrid nonlinear two-level Schwarz methods and energy minimizing coarse spaces for unstructured grids. SIAM Journal on Scientific Computing, 42(4), A2461-A2488.

Heinlein, A., Klawonn, A., Rajamanickam, S., and Rheinbach, O. (2020). FROSch: A fast and robust overlapping Schwarz domain decomposition preconditioner based on Xpetra in Trilinos (pp. 176-184). Springer International Publishing.