Project: Stabilization Methods for Physics-Informed Neural Networks Description:

Various disciplines such as aerodynamics, structural mechanics, propulsion technology, and control systems must be integrated in aircraft designs. Creating an initial rough draft of a new aircraft design considering the various disciplines should not require extensive time and effort. To enable rapid evaluation of the effects of design changes across several disciplines **surrogate models** are used for efficient multidisciplinary optimization.

Physics-informed neural networks (PINNs) are powerful surrogate models that combine the principles of

physics with neural network (NN) models [1]. One ansatz to integrate physical principles into NNs is the integration of numerical methods such as the **finite element method** (FEM). Using FEM-based NNs [2] gives rise to similar challenges as with the classical FEM. These can be solved with the help of techniques from numerical analysis.

For **flow problems** dominated by advection (e.g. flow problems with high Reynolds numbers), standard FEM can lead to oscillating and inaccurate solutions. Stabilization techniques, such as Streamline-Upwind/Petrov-Galerkin (SUPG) help to dampen these oscillations and generate **stable solutions** [3]. In the SUPG method, an additional stabilization term is added to the weak formulation of the differential equation. This stabilization term considers the gradient of the solution in the direction of flow and helps to dampen numerical oscillations.



Figure 1: Stokes velocity field around an airfoil with an angle of attack of 5 degree calculated from a FEM-based NN.

The goal of this master project is to **investigate if stabilization techniques** as SUPG can **help FEM-based NNs** to **predict stable solution** for **incompressible flow problems**. This is a project in collaboration with the German Aerospace Center (DLR), which means that existing code for FEM-based NNs developed at DLR can be used.

Tasks:

- 1. Familiarize with the Finite Element Method and stabilization techniques.
- 2. Familiarize with the basic concepts of physics-informed neural networks, especially FEM-based NNs.
- 3. Implement stabilization techniques as Streamline-Upwind/Petrov-Galerkin (SUPG) for the existing FEM-based NNs using PyTorch¹.
- 4. Train and test the predictions of the stabilized FEM-based NNs.

Contact:

If you are interested in this project and/or have further questions please contact Alexander Heinlein, <u>a.heinlein@tudelft.nl</u>, Franziska Griese, <u>Franziska.Griese@dlr.de</u>, and Philipp Knechtges <u>Philipp.Knechtges@dlr.de</u>.

^{1 &}lt;u>https://pytorch.org/</u>

References:

[1] M. Raissi, P. Perdikaris, G.E. Karniadakis. *Physics-informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations*. Journal of Computational Physics, Vol. 378, pp. 686-707, 2019.

[2] Electronic library - Solving Stokes Flow with Hybrid ML-Simulation Methods (dlr.de)

[3] T. E. Tezduyar. Stabilized Finite Element Formulations for Incompressible Flow Computations. *Advances in Applied Mechanics*, Elsevier, Vol. 28, pp. 1–44, 1991. <u>doi:10.1016/S0065-2156(08)70153-4</u>.