# Approximately well-balanced Discontinuous Galerkin methods using bases enriched with Physics-Informed Neural Networks

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This work concerns the enrichment of Discontinuous Galerkin (DG) bases, so that the resulting scheme provides a much better approximation of steady solutions to hyperbolic systems of balance laws. The basis enrichment leverages a prior – an approximation of a family of steady solutions – which we propose to compute using a parametric Physics-Informed Neural Network (PINN). To that end, after presenting the classical DG scheme, we show how to enrich its basis with a prior. Convergence results and error estimates follow, in which we prove that the basis with prior does not change the order of convergence, and that the error constant is improved. To construct the prior, we elect to use parametric PINNs, which we introduce, as well as the algorithms to construct a prior from PINNs. We finally perform several validation experiments on four different hyperbolic balance laws to highlight the properties of the scheme. Namely, we show that the DG scheme with prior is much more accurate on steady solutions than the DG scheme without prior, while retaining the same approximation quality on unsteady solutions.