Hybrid Finite Element / Neural Network simulations

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We present a hybrid simulation method that combines a coarse mesh finite element method with local fluctuations that are learned in a deep neural network. The general idea of the approach is that classical tools like finite elements or finite volumes are highly efficient and reliable but not fully satisfactory when it comes to resolving all scales of a complex problem.

Our approach corrects the coarse solution locally: this means that the neural network - opposed to classical PINNs - does not represent the entire solution of the differential equations, but acts only on very small subsets of the domain that we call patches. Hereby, we get efficiency and also good generalizability.

For the 3D Navier-Stokes equations we show substantial computational savings. Furthermore, we demonstrate first steps of an error estimate that allows to control the hybrid discretization error in a mixture of a priori and a posteriori estimate.