Nonlinear joint spectral radius of cone order preserving functions

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A Neural Network can be seen as a discrete switched system that alternates maps from a class of nonlinear functions following a switching rule that is not known a-piori, but is determined by the training. In particular, stability properties of the NN can be studied in terms of the stability of the corresponding nonlinear switched system. Motivated by applications in machine learning, we study the stability of nonlinear switched systems that alternate homogeneous nonlinear functions that preserve the ordering induced by a cone. Such maps, admitting the notion of spectral radius, allow us to generalize, from the linear to the nonlinear case, the study of the joint spectral radius (JSR) of the system. In particular, we first prove that the value of the JSR yields information about the stability of the system. Hence, we investigate the properties of the nonlinear JSR, tracing analogies and differences from the linear case. Finally, we present an algorithm devoted to computing the nonlinear JSR.

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