An introduction to the lowest-order Neural Approximated Virtual Element Method

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Thanks to the rapidly growing interest in Scientific Machine Learning, numerous numerical methods relying on deep neural networks have been proposed by the scientific community in the last few years. One of such methods is the Neural Approximated Virtual Element Method (NAVEM), a polygonal method to solve partial differential equations (PDEs) recently introduced in [1]. The method combines the nonlinear properties and the efficiency of a neural network with the flexibility and the accuracy of the more classical Virtual Element Method (VEM) [2].

The VEM is a polygonal method relying on virtual functions, i.e. functions that are not known in a closed form. Therefore, suitable polynomial projectors and stabilization terms are necessary to evaluate the differential operators characterizing the PDE and to retrieve the coercivity of the discrete problem. The NAVEM replaces such unknown virtual basis functions by suitable neural network-based approximations with similar functional properties. This way, it is possible to define a new polygonal method without any projection or stabilization, which are problem-dependent and may limit the model accuracy in case of strongly anisotropic PDEs.

In order to approximate the local VEM basis functions, we employ linear combinations of suitable basis functions, where the coefficients are predicted by a deep neural network. This way, it is also possible to reduce spurious oscillations and discontinuities in the local and global basis functions respectively, and improve the NAVEM stability as a consequence. Two-dimensional numerical tests on distorted quadrilateral meshes and on Voronoi meshes are proposed to validate the method accuracy and flexibility.

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References

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